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Flow-gauge Slavnov-Taylor identities for Zwanziger's gauge fixing

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The generalization of the Slavnov-Taylor identities for the stochastically quantized Yang-Mills field theory with either Zwanziger gauge fixing or, equivalently, Faddeev-Popov flow-gauge fixing in one higher dimension is presented. Those exact relationships among Green's functions in the stochastically quantized theory are derived by extending suitably Slavnov's method. As a consequence there is no renormalization of the longitudinal part of Green's functions in $\alpha=0$, to all perturbative orders. Based on the general identities, the divergent longitudinal part of the two-point Green's function is calculated to second order for $\alpha=1$, and it is found to agree with other independent calculations.

One of the gaps to be filled in accomplishing the program of stochastic quantization of non-Abelian Yang-Mills field theory[1,2] with Zwanziger's gauge-fixing term[3] is the obtainment of the analogues of the Slavnov-Taylor identities.[4] Another gap, among others, is to use them, as much as possible, to get dynamical results which generalize those derived in the standard Faddeev-Popov case through, for instance, Slavnov's method.[4] In fact, the lack of such identities has been mentioned repeatedly in Refs. 2 and 5. In this paper we derive such identities by exploiting the equivalence of Zwanziger's gauge fixing with Faddeev-Popov flow-gauge fixing,[6,7] in one higher dimension and make use of them later. Further details concerning these gauges and the equivalence are found in Ref. 6.

There have been other approaches in this direction: in Ref. 8, the Ward identities associated with the background gauge invariance of the generating functional were derived and in Ref. 5 the stochastic gauge-fixing terms is not considered, so that only gauge-invariant quantities were treated. Other previous works[9] have used a different symmetry of the stochastic action: namely, its so-called hidden supersymmetry associated with the stochastic time-reversal invariance. The related identities have been used, for instance, to prove the renormalizability of the stochastically quantized theory.[10] On the other hand, we must point out that this supersymmetry is harder to analyze when the stochastic gauge-fixing term is considered.

Following Ref. 6 we begin with the following five-dimensional gauge theory, in which we have included a diffusion parameter γ:

$$\mathcal{L}_5=\frac{1}{4\gamma}\left[F^a_{5\mu}+\gamma\frac{\delta S_{\rm YM}}{\delta A^a_\mu}\right]^2-\frac{1}{2}\frac{\delta^2 S_{\rm YM}}{\delta A^a_\mu\delta A^a_\mu}\ , \tag{1}$$

whose gauge invariance is maintained with the five-dimensional field strength

$$F^a_{5\mu}=\partial_t A^a_\mu-\gamma D^{ab}_\mu A^b_5\ . \tag{2}$$

$S_{\rm YM}$ is the usual Euclidean Yang-Mills action in four dimensions and D^{ab}_μ is the covariant derivative, namely,

$$S_{\rm YM}=\tfrac{1}{4}\int d^Dx\, F^a_{\mu\nu}F^a_{\mu\nu}\ , \tag{3}$$

$$D^{ab}_\mu=\partial_\mu\delta^{ab}-gf^{abc}A^c_\mu\ . \tag{4}$$

The flow gauges as a class are ghostless and infrared soft, and the particular choice

$$A^b_5=\frac{1}{\alpha}\partial_\mu A^b_\mu=v^b \tag{5}$$

corresponds to the usual Zwanziger gauge-fixing function. Indeed, the Zwanziger gauge-fixed Langevin equation

$$\partial_t A^a_\mu=-\gamma\frac{\delta S_{\rm YM}}{\delta A^a_\mu}+\gamma D^{ab}_\mu v^b+\eta^a_\mu \tag{6}$$

is the Nicolai map of the $\mathcal{L}_5$ with this particular flow-gauge choice. $\eta^a_\mu(x,t)$ in (6) is the Gaussian correlated noise.

The starting point of our analysis is the generating functional for the flow-gauge-fixed five-dimensional gauge theory $[J^a_\mu=J^a_\mu(x,t)]$

$$Z[J]=N\int[DA]\exp\left[\int dt\, d^Dx(-\mathcal{L}_5+J^a_\mu A^a_\mu)\right]\ , \tag{7}$$

which was also found in Ref. 11 using the Fokker-Planck equation associated with (6). In order to extract from (7) the flow-gauge Slavnov-Taylor identities we shall extend suitably Slavnov's method[4] to (7). That is, we perform in (7) a functional change of integration variables which is given, precisely, by an infinitesimal gauge transformation depending also on the fifth time:

$$A^a_\mu\to A^a_\mu+D^{ab}_\mu\theta^b(x,t)\ , \tag{8}$$

$\theta^b(x,t)$ being the infinitesimal parameters of the gauge transformations. A similar transformation with a t-independent θ parameter was considered in Ref. 8, in order to show the usefulness of the background field method, rather than to derive the nonperturbative Slavnov-Taylor identities as is our purpose here.

Since $Z[J]$ in (7) remains unchanged under (8) and by expanding the exponential in (7) to the first order in $\theta^b(x,t)$ we arrive, after some algebra, at the following relation:

$$0=\int [DA]\exp\left[\int dt\, d^Dx(-\mathcal{L}_5+J^a_\mu A^a_\mu)\right] \times\int dt\, d^Dx\left\{J^a_\mu D^{ab}_\mu\theta^b-\frac{1}{2\gamma}\left[\partial_t A^a_\mu+\gamma\frac{\delta S_{\rm YM}}{\delta A^a_\mu}-\gamma D^{ad}_\mu v^d\right]\right.$$
$$\times\left[\partial_t D^{ab}_\mu\theta^b+\gamma g f^{abc}\frac{\delta S_{\rm YM}}{\delta A^b_\mu}\theta^c+\gamma g f^{abc}v^b D^{ce}_\mu\theta^e-\gamma D^{ab}_\mu\left[\frac{\delta v^b}{\delta A^c_\nu}(D^{ce}_\nu\theta^e)\right]\right]$$
$$\left.+g^2C_2(G)(\delta_{\mu\mu}-1)\delta^{(D)}(0)(\partial_\nu A^b_\nu)\theta^b\right\}. \tag{9}$$

We introduce in (9) the explicit Zwanziger's gauge fixing (5) and extract the infinitesimal gauge transformation parameter $\theta^b(x,t)$ as a common factor. After some nontrivial algebra, the resulting expression is the general stochastic Slavnov-Taylor identity for any J^a_μ:

$$0=\int [DA]\exp\left[\mathcal{L}_5+\int dt\, d^Dx\, J^a_\mu A^a_\mu\right]\left\{-D^{ba}_\mu\left[J^a_\mu+\frac{1}{2\gamma}\left[\delta_{\mu\nu}\delta^{ad}\partial_t+\frac{\gamma}{\alpha}\partial_\mu D^{ad}_\nu-\frac{\gamma}{\alpha}gf^{dca}\delta_{\mu\nu}(\partial_\rho A^c_\rho)\right]\right.\right.$$
$$\left.\left.\times\left[\delta_{\nu\sigma}\delta^{de}\partial_t-\frac{\gamma}{\alpha}D^{de}_\nu\partial_\sigma\right]A^e_\sigma\right]\right\}. \tag{10}$$

Remarkably, (10) has turned out to become independent of $S_{\rm YM}$, as expected, however, for Slavnov-Taylor identities. Nevertheless, its maximum order in g, in the term multiplying the exponential in (10), is g^3 (terms of higher orders in g have canceled by symmetry properties in the color indices). Thus, the identities are more complicated than the usual ones found within the four-dimensional Faddeev-Popov gauge-fixing procedure. As we will use systematically dimensional regularization throughout this paper, we have dropped the term proportional to $\delta^{(D)}(0)$ in (9) that acts as a counterterm.[10] When $J=0$, (10) yields an identity between complete Green's functions of one, two, three, and four points (that is, including disconnected terms and pieces which are not one-particle irreducible).

We now turn to verify explicitly (10) in $g=0$. In this limit $Z[J]=Z_0[J]$ and Eq. (10) is seen to become

$$0=-\partial_\mu J^a_\mu Z_0[J]+\left[-\frac{1}{2\gamma}\frac{\partial^2}{\partial t^2}+\frac{\gamma}{2\alpha^2}\partial_\nu\partial_\nu\partial_\sigma\partial_\sigma\right]\partial_\mu\left[\frac{\delta Z_0[J]}{\delta J^a_\mu}\right], \tag{11}$$

where $Z_0[J]$ can be expressed after performing a Gaussian integration as

$$Z_0[J]=\exp\left[\tfrac{1}{2}\int dt_1dt_2\int d^Dx_1d^Dx_2J^a_\mu(x_1,t_1)G^{ab(0)}_{\mu\nu}(x_1-x_2,t_1-t_2)J^b_\nu(x_2,t_2)\right]. \tag{12}$$

$G^{ab(0)}_{\mu\nu}(x_1-x_2,t_1-t_2)$ is the free propagator that has been previously given in Ref. 11. Some algebra shows that (12) is identically verified for any $J^a_\mu(x,t)$ and any gauge parameter α.

We shall extract from (10) a specific nontrivial identity which generalizes one obtained by Slavnov.[4] We differentiate (10) with respect to $J^f_\rho(y,t')$ and take $J=0$. The result is

$$0=-\langle D^{bf}_\rho\delta(x-y)\delta(t-t')\rangle-\left\langle\frac{1}{2\gamma}A^f_\rho(y,t')D^{ba}_\mu\partial_t^2A^a_\mu(x,t)\right\rangle+\left\langle\frac{1}{2\alpha}A^f_\rho(y,t')D^{ba}_\mu\partial_tD^{ad}_\mu\partial_\nu A^d_\nu(x,t)\right\rangle$$
$$-\left\langle\frac{1}{2\alpha}A^f_\rho(y,t')D^{ba}_\mu\partial_\mu D^{ad}_\nu\partial_tA^d_\nu(x,t)\right\rangle+\left\langle\frac{\gamma}{2\alpha^2}A^f_\rho(y,t')D^{ba}_\mu\partial_\mu D^{ad}_\nu D^{de}_\nu\partial_\sigma A^e_\sigma(x,t)\right\rangle$$
$$+\left\langle\frac{1}{2\alpha}gf^{adc}A^f_\rho(y,t')D^{ba}_\mu(\partial_\nu A^c_\nu(x,t))\partial_tA^d_\mu(x,t)\right\rangle-\left\langle\frac{\gamma}{2\alpha^2}gf^{adc}A^f_\rho(y,t')D^{ab}_\mu(\partial_\nu A^c_\nu(x,t))D^{de}_\mu\partial_\sigma A^e_\sigma(x,t)\right\rangle, \tag{13}$$

where $\langle H(A)\rangle$ represents the corresponding correlation function of $H(A)$. Equation (13) is more readable in momentum-energy space. In fact, by Fourier transforming (13) we get

$$0=\delta^{(D)}(p+p')\delta(\omega+\omega')\left\{ip_\rho\delta^{fb}+gf^{bfd}G^d_\rho(0,0)-\frac{1}{2\gamma}ip_\mu\left[\omega^2+\frac{\gamma^2}{\alpha^2}(p^2)^2\right]G^{fb}_{\rho\mu}(p,\omega)\right.$$
$$+\frac{\gamma g}{2\alpha^2}f^{bad}\int\frac{d^Dq\,D\Omega}{(2\pi)^{D+1}}\left[-\frac{\alpha^2}{\gamma^2}(\omega+\Omega)^2\delta_{\mu\nu}-i\frac{\alpha}{\gamma}p_\nu(p+q)_\mu\omega-i\frac{\alpha}{\gamma}(\omega+\Omega)p_\mu q_\nu\right.$$

$$+i\frac{\alpha}{\gamma}p^2(\omega+\Omega)\delta_{\mu\nu}+p_\sigma q_\nu(p+q)_\sigma(p+q)_\mu$$

$$\left.-p^2(2p+q)_\nu(p+q)_\mu-(p+q)^2(p+q)_\nu(p+q)_\mu\right]$$

$$\times G^{fda}_{\rho\nu\mu}(p,\omega;q,\Omega;-p-q,-\omega-\Omega)$$

$$+\frac{\gamma g^2}{2\alpha^2}f^{bad}f^{ace}\int\frac{d^Dq\,d\Omega\,d^Dq'd\Omega'}{(2\pi)^{2D+2}}\left[\frac{\alpha}{\gamma}(\omega+\Omega+\Omega')[(p+q)_\mu\delta_{\nu\sigma}-q'_\nu\delta_{\mu\sigma}]\right.$$

$$+\left[-\frac{\alpha}{\gamma}(\omega+\Omega)\delta_{\mu\nu}+ip^2\delta_{\mu\nu}-ip_\nu q_\mu\right.$$

$$\left.\left.-iq'_\nu q'_\mu+2i(p+q)_\mu(p+q)_\nu\right](p+q+q')_\sigma\right]$$

$$\times G^{fdec}_{\rho\mu\nu\sigma}(p,\omega;q,\Omega;q',\Omega';-p-q-q',-\omega-\Omega-\Omega')$$

$$-\frac{\gamma g^3}{2\alpha^2}f^{aed}f^{bdg}f^{ach}\int\frac{d^Dq\,d\Omega\,d^Dq'd\Omega'd^Dq''d\Omega''}{(2\pi)^{3D+3}}\{[q'_\nu\delta_{\mu\sigma}-(p+q)_\mu\delta_{\nu\sigma}](p+q+q'+q'')_\epsilon]\}$$

$$\left.\times G^{fgehc}_{\rho\mu\nu\sigma\epsilon}(p,\omega;q,\Omega;q',\Omega';q'',\Omega'';-p-q-q'-q'',-\omega-\Omega-\Omega'-\Omega'')\right\}, \tag{14}$$

where $G^{a_1\cdots a_n}_{\mu_1\cdots\mu_n}$ are the n-point Green's functions in Fourier space.

This exact flow-gauge identity is the generalization, to the stochastic quantization formulation including the stochastic gauge-fixing term (5), of one of the Slavnov-Taylor identities for non-Abelian Yang-Mills theory.

It has been stated in Ref. 5 that the actual form of the identities is the same as in the Faddeev-Popov quantization, the only difference being in the stochastic Green's functions themselves. It is to be remarked that this result was obtained without introducing the Zwanziger gauge-fixing term, that seems unavoidable in computing noninvariant gauge quantities, so that it holds only when all the quantities related through the identities are gauge invariant.

The relation of Eq. (14) to the one derived from Faddeev-Popov quantization (see Refs. 4 and 12 for instance) may be obtained multiplying (14) by

$$2\gamma\Big/\left[\omega^2+\frac{\gamma^2}{\alpha^2}(p^2)^2\right]$$

and integrating over ω. When these operations are carried out, we see that the Slavnov-Taylor identity[4,12] for the two-point function is modified by the contributions coming from the terms in (14) that involve three-, four-, and five-point Green's functions, respectively, as follows:

$$0=\alpha\frac{p_\rho}{p^2}\delta^{fb}-p_\mu G^{fb}_{\rho\mu}(p)+D^{fb,(3)}_\rho(p)+D^{fb,(4)}_\rho(p)$$

$$+D^{fb,(5)}_\rho(p)\ , \tag{15}$$

where

$$G^{fb}_{\rho\mu}(p)=\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}G^{fb}_{\rho\mu}(p,\omega)$$

is the equilibrium two-point function and $D^{fb,(3)}_\rho(p)$, $D^{fb,(4)}_\rho(p)$, and $D^{fb,(5)}_\rho(p)$ are the corrections due to the three-, four-, and five-point functions, respectively. Note that the tadpole contribution of $G^d_\rho(0,0)$ may be safely disregarded in (14) as can be easily proved by symmetry arguments in the color indices or dimensional regularization.

There is a specific choice of the gauge parameter α, namely, $\alpha\to 0$, for which we have been able to perform the integrals over ω and Ω in $D^{fb,(3)}_\rho(p)$, $D^{fb,(4)}_\rho(p)$, and $D^{fb,(5)}_\rho(p)$. In this limit, the equilibrium Green's functions are conjectured[13] to approach the Landau gauge Faddeev-Popov results, as has been shown to the one-loop order in Refs. 2 and 14. Assuming finiteness of Green's functions as $\alpha\to 0$ (Ref. 15), the result is

$$0=-p_\rho p^2G^{fb}_{\rho\mu}(p)+igf^{bad}\int\frac{d^Dq}{(2\pi)^D}[p_\sigma(p+q)_\sigma q_\nu-p^2(2p+q)_\nu-(p+q)^2(p+q)_\nu](p+q)_\mu G^{fda}_{\rho\nu\mu}(p,q,-p-q)$$

$$-g^2f^{bad}f^{ace}\int\frac{d^Dq\,d^Dq'}{(2\pi)^{2D}}[p^2\delta_{\mu\nu}-p_\nu q_\mu-q'_\nu q'_\mu+2(p+q)_\mu(p+q)_\nu](p+q+q')_\sigma G^{fdec}_{\rho\mu\nu\sigma}(p,q,q',-p-q-q')$$

$$-ig^3 f^{aed} f^{bdg} f^{ach} \int \frac{d^D q\, d^D q'\, d^D q''}{(2\pi)^{3D}} [(q'_\nu \delta_{\mu\sigma} - (p+q)_\mu \delta_{\nu\sigma})](p+q+q'+q'')_\epsilon$$
$$\times G^{fgehc}_{\rho\mu\nu\sigma\epsilon}(p,q,q',q'',-p-q-q'-q'') \ . \tag{16}$$

It is not difficult to show that the usual Faddeev-Popov result, namely, the transversality of the renormalized propagator in the Landau gauge, is contained in (16). Similar D-dimensional Ward identities are derived in Ref. 15 by other methods. In fact, the contributions coming from the three-, four-, and five-point Green's functions in (16) vanish when the propagator is assumed transverse. For instance, the three-point Green's function may be written as

$$G^{fda}_{\rho\nu\mu}(p,q,-p-q) = G^{ff'}_{\rho\rho'}(p) G^{dd'}_{\nu\nu'}(q) G^{aa'}_{\mu\mu'}(-p-q) \times \Gamma^{f'd'a'}_{\rho'\nu'\mu'}(p,q,-p-q) \tag{17}$$

and the right-hand side vanishes when contracted with $(p+q)_\mu$ as in (16), because we are assuming $G^{aa'}_{\mu\mu'}(-p-q)$ to be transverse. The same argument holds for a generic N-point Green's function. This is not true for other choices of α, as may be seen in (15).

Thus, for the gauge parameter choice $\alpha=0$, we have shown that there is no renormalization of the longitudinal part of Green's functions in the equilibrium limit to all orders. This is precisely the stochastic analogue in $\alpha=0$ of Slavnov's result for the two-point function [see Eq. (23) in Ref. 4]. Accordingly, the counterterm of the mass renormalization for $\alpha=0$ will vanish.

We have also calculated the divergent part of $D^{fb,(3)}_\rho$ to the total order g^2 (those of $D^{fb,(4)}_\rho$ and $D^{fb,(5)}_\rho$ are of higher orders in g; we should have considered also nonconnected contributions, to total order g^2, coming from the four-point Green's function. However, it is not difficult to show that they cancel when dimensional regularization is used): this amounts to integrating the three-leg vertex at the tree level. Some lengthy calculations show that such divergent contribution to $D^{fb,(3)}_\rho$ is

$$D^{fb,(3)}_\rho(p) = \frac{(g/2)^2 C_2(G)\delta^{fb}}{(4\pi)^2(-\epsilon)} \frac{7}{2} \frac{p_\rho}{p^2} + \text{finite parts} + O(g^3) \ , \tag{18}$$

where $\epsilon=(4-D)/2$, and $C_2(G)$ is equal to N for a gauge group $G=\mathrm{SU}(N)$. Comparing with Refs. 2 and 14 we see that (17) equals the longitudinal part of the propagator that arises in stochastic quantization to the one-loop order in the gauge ($\alpha=1$).

The identities derived in (14) and (15) are, as we have shown, useful in order to analyze longitudinal parts of dynamical Green's functions, and, so, they are the stochastic counterparts of some of Slavnov's ones, but it is not, as yet, clear whether they may give relations between renormalization constants that would ensure the renormalizability of the theory. It seems that, in order to accomplish the latter task, we should derive more complicated identities, by essentially taking more derivatives of (10). This rather complicated task lies beyond the scope of the present work.

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[1]G. Parisi and Y. Wu, Sci. Sin. **24**, 483 (1981); E. Floratos and J. Iliopoulos, Nucl. Phys. **B214**, 392 (1983); N. Namiki, I. Ohba, K. Okano, and Y. Yamanaka, Prog. Theor. Phys. **69**, 1580 (1983).
[2]Z. Bern, M. B. Halpern, L. Sadun, and C. Taubes, Nucl. Phys. **B284**, 1 (1987); **B284**, 35 (1987); Z. Bern, M. B. Halpern, and L. Sadun, *ibid.* **B284**, 92 (1987); Z. Bern, H. S. Chan, and M. B. Halpern, Z. Phys. C **33**, 77 (1986).
[3]D. Zwanziger, Nucl. Phys. **B192**, 259 (1981).
[4]A. A. Slavnov, Theor. Math. Phys. **10**, 99 (1972).
[5]S. Pugnetti, Phys. Lett. B **183**, 465 (1987).
[6]H. S. Chan and M. B. Halpern, Phys. Rev. D **33**, 540 (1986).
[7]H. Chen and E. C. Tsai, Phys. Rev. D **34**, 3858 (1986); F. Steiner, Phys. Lett. B **173**, 321 (1986).
[8]K. Okano, Nucl. Phys. **B289**, 109 (1987).
[9]G. Parisi and N. Sourlas, Nucl. Phys. **B206**, 321 (1982); B. McClain, A. Niemi, and C. Taylor, Ann. Phys. (N.Y.) **140**, 232 (1982); E. S. Egorian and S. Kalitsin, Phys. Lett. **129B**, 320 (1983); H. Nakazato, M. Namiki, I. Ohba, and K. Okano, Prog. Theor. Phys. **70**, 298 (1983); E. Gozzi, Phys. Rev. D **28**, 1922 (1983); **30**, 1218 (1984).
[10]J. Zinn-Justin, Nucl. Phys. **B275** [FS17], 135 (1986).
[11]R. F. Alvarez-Estrada and A. Muñoz Sudupe, J. Phys. A **20**, 733 (1987).
[12]P. Pascual and R. Tarrach, *QCD: Renormalization for the Practitioner* (Lecture Notes in Physics, Vol. 194) (Springer, Berlin, 1984).
[13]D. Zwanziger, Phys. Lett. **141B**, 337 (1982).
[14]A. Muñoz Sudupe and R. F. Alvarez-Estrada, Phys. Lett. **164B**, 102 (1985); A. Muñoz Sudupe and L. A. Fernández, Phys. Rev. D **36**, 510 (1987).
[15]M. B. Halpern, Report No. LBL-24141 (unpublished).